

COMPARISON OF ASTEROIDS OBSERVED IN THE SLOAN DIGITAL SKY SURVEY¹ WITH A CATALOG OF KNOWN ASTEROIDS

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ABSTRACT

We positionally correlate asteroids from existing catalogs with a sample of $\sim 18,000$ asteroids detected by the Sloan Digital Sky Survey (SDSS, Ivezić *et al.* 2001). We find 2641 unique matches, which represent the largest sample of asteroids with both accurate multi-color photometry and known orbital parameters. The matched objects are predominantly bright, and demonstrate that the SDSS photometric pipeline recovers $\sim 90\%$ of the known asteroids in the observed region. For the recovered asteroids we find a large offset (~ 0.4 mag) between Johnson V magnitudes derived from SDSS photometry and the predicted catalog-based visual magnitudes. This offset varies with the asteroid color from 0.34 mag for blue asteroids to 0.44 mag for red asteroids, and is probably caused by the use of unfiltered CCD observations in the majority of recent asteroid surveys. This systematic photometric error leads to an overestimate of the number of asteroids brighter than a given absolute magnitude limit by a factor of ~ 1.7 . The distribution of the matched asteroids in orbital parameter space indicates strong color segregation. We confirm that some families are dominated by a single asteroid type (e.g. the Koronis family by red asteroids and the Themis family by blue asteroids), while others appear to be a mixture of blue and red objects (e.g. the Nysa/Polana family). Asteroids with the bluest $i^* - z^*$ colors, which can be associated with the Vesta family, show particularly striking localization in orbital parameter space.

Subject headings: Solar system - asteroids

1. INTRODUCTION

The Sloan Digital Sky Survey (SDSS) is a digital photometric and spectroscopic survey which will cover $10,000 \text{ deg}^2$ of the Celestial Sphere in the North Galactic cap and produce a smaller ($\sim 225 \text{ deg}^2$) but much deeper survey in the Southern Galactic hemisphere (York *et al.* 2000 and references therein). The survey sky coverage will result in photometric measurements for about 50 million stars and a similar number of galaxies. The flux densities of detected objects are measured almost simultaneously in five bands (u , g , r , i , and z ; Fukugita *et al.* 1996) with effective wavelengths of $3551 r^*A$, $4686 r^*A$, $6166 r^*A$, $7480 r^*A$, and $8932 r^*A$, 95% complete¹⁴ for point sources to

limiting magnitudes of 22.0, 22.2, 22.2, 21.3, and 20.5 in the North Galactic cap¹⁵. Astrometric positions are accurate to about 0.1 arcsec per coordinate (rms) for sources brighter than 20.5^m (Pier *et al.* 2001), and the morphological information from the images allows robust star-galaxy separation to $\sim 21.5^m$ (Lupton *et al.* 2002).

SDSS, although primarily designed for observations of extragalactic objects, will significantly contribute to studies of the solar system objects, because asteroids in the imaging survey must be explicitly detected to avoid contamination of the samples of extragalactic objects selected for spectroscopy. Ivezić *et al.* (2001, hereafter I01) analyzed SDSS commissioning data and showed that SDSS

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¹⁴These values are determined by comparing multiple scans of the same area obtained during the commissioning year. Typical seeing in these observations was 1.5 ± 0.1 arcsec.

¹⁵We refer to the measured magnitudes in this paper as u^* , g^* , r^* , i^* , and z^* because the absolute calibration of the SDSS photometric system (dependent on a network of standard stars) is still uncertain at the $\sim 0.03^m$ level. The SDSS filters themselves are referred to as u , g , r , i , and z . All magnitudes are given on the AB_V system (Oke & Gunn 1983, for additional discussion regarding the SDSS photometric system see Fukugita *et al.* 1996, and Stoughton *et al.* 2002).

will increase the number of asteroids with accurate five-color photometry by more than two orders of magnitude (to about 100,000), and to a limit about seven magnitudes fainter than previous multi-color surveys (e.g. The Eight Color Asteroid Survey by Zellner, Tholen & Tedesco 1985). The main results derived from these early SDSS observations are

1. A measurement of the main-belt asteroid size distribution to a significantly smaller size limit (~ 1 km) than possible before. The size distribution resembles a broken power-law, independent of the heliocentric distance: $D^{-2.3}$ for $0.4 \text{ km} \lesssim D \lesssim 5 \text{ km}$, and D^{-4} for $5 \text{ km} \lesssim D \lesssim 40 \text{ km}$.
2. A smaller number of asteroids compared to previous work. In particular, the number of asteroids with diameters larger than 1 km is about 7×10^5 , or up to three times less than suggested by earlier studies.
3. The distribution of main-belt asteroids in 4-dimensional SDSS color space is strongly bimodal, and the two groups can be associated with S (rocky) and C (carbonaceous) type asteroids. A strong bimodality is also seen in the heliocentric distribution of asteroids: the inner belt is dominated by S type asteroids centered at $R \sim 2.8 \text{ AU}$, while C type asteroids, centered at $R \sim 3.2 \text{ AU}$, dominate the outer belt.

I01 demonstrated that the SDSS photometric pipeline (*photo*, Lupton *et al.* 2002) is a robust and highly efficient automated tool for finding moving objects. I01 present a detailed discussion of the completeness and reliability of the SDSS asteroid catalog based on the known SDSS photometric and astrometric precision, but this discussion is incomplete because it does not attempt a direct comparison with catalog of known asteroids on an object-by-object basis. The importance of such an analysis is evident from the realization that there are $\sim 90,000$ *cataloged* asteroids with $H < 15.5$, while I01 expect to find only $\sim 58,000$ in the same absolute magnitude¹⁶ range. The implied low completeness of 64% (compare to 98% estimated by I01) can only be verified by direct matching of known and SDSS asteroids¹⁷.

Another motivation for cross-correlating SDSS asteroids and known asteroids is a large potential increase in the number of asteroids with both accurate multi-color photometry and known orbital parameters (the SDSS data themselves are insufficient for accurate orbit determination, but will provide serendipitous photometric measurements for a substantial fraction of known asteroids). Additionally, the SDSS color information may be utilized to study the chemical segregation in the full orbital parameter space, rather than only as a function of heliocentric distance as done by I01.

This paper presents the first results on cross-correlating SDSS asteroids and cataloged asteroids. Section 2 describes the analyzed data, a software pipeline used for

generating the positions of known asteroids at the time of SDSS observations, and their positional matching to objects automatically detected by the SDSS photometric pipeline. Section 3 discusses the statistics of matched objects, and compares the SDSS photometric measurements with the apparent magnitudes predicted from cataloged absolute magnitudes. In Section 4 we discuss the chemical segregation of asteroids in orbital element space, and in Section 5 we summarize the main results.

2. THE MATCHING OF SDSS MOVING OBJECTS AND KNOWN ASTEROIDS

2.1. SDSS data

We use asteroid data (c.f. I01) from the SDSS Early Data Release, described in detail by Stoughton *et al.* 2002 (hereafter SDSSedr). These data include equatorial observing runs 94, 125, 752 and 756 (see I01 and SDSSedr); the boundaries are given by $-1.27^\circ \lesssim \text{Dec} \lesssim 1.27^\circ$, and $RA = 351^\circ - 56^\circ$ (runs 94 and 125), or $RA = 145^\circ - 250^\circ$ (runs 752 and 756). For a footprint in ecliptic coordinates, see Figure 1 in I01. This region has an area of 432 deg^2 and includes 12,668 moving objects selected as in I01 (the velocity $v > 0.03 \text{ deg/day}$ and $14.0 < r^* < 21.5$). We use this sample for all quantitative estimates of the catalog completeness, and for photometric comparison. In order to increase the number of objects when studying the color segregation in the asteroid belt (Section 4), we also add additional objects from seven currently unreleased equatorial observing runs¹⁸ that roughly double the matched sample size.

The SDSS photometric data include a list of objects flagged as moving by the photometric pipeline. For each object, the position and time of observation, and the magnitudes and associated errors in five SDSS photometric bands (u, g, r, i, z) are recorded. The SDSS imaging data is obtained in the time-delay-and-integrate (TDI, or “drift-scanning”) mode¹⁹, and thus each observed position corresponds to a different observing time (as opposed to the starring mode where all objects from a given image are observed at the same time). In a general case, the correlation between a position and time is easiest to compute in the great circle coordinate system. Since all the scans discussed here were obtained along the Celestial Equator, this dependence becomes particularly simple, and is given by

$$T(RA) = T_0 + \frac{RA - RA_0}{360^\circ} \text{days}, \quad (1)$$

where T is the time corresponding to position RA (e.g. Julian Day), and the zeropoints T_0 and RA_0 are constants for a given run (all reported positions correspond to the r band). For an arbitrary run, this expression is easily generalized by using an appropriate coordinate system.

The cataloged asteroid magnitudes are reported in the Johnson V band. Preliminary transformations from the SDSS photometric system to the Johnson bands are given by Fukugita *et al.* (1996) and Krisciunas, Margon &

¹⁶The absolute magnitude is the magnitude that an asteroid would have at a distance of 1 AU from the Sun and from the Earth, viewed at zero phase angle. For more details see I01 and references therein.

¹⁷We thank R. Jedicke for pointing out this discrepancy to us.

¹⁸Prompt release of asteroid data (i.e. without any waiting period) is currently under consideration by SDSS.

¹⁹See <http://www.astro.princeton.edu/PBOOK/welcome.htm>

Szkody (1998). We use a recent updated version of these transformations (M. Fukugita, priv. comm.; the new transformations produce V magnitudes that agree to better than 0.1 mag with the version from Fukugita *et al.* 1996) to synthesize the Johnson B and V band magnitudes²⁰

$$\begin{aligned} V_o &= r^* + 0.44(g^* - r^*) - 0.02 \\ (B - V)_o &= 1.04(g^* - r^*) + 0.19 \end{aligned} \quad (2)$$

(3)

For typical values of $g^* - r^*$ for asteroids (0.4–0.8), $V_o - r^*$ is in the range 0.16 to 0.33. The overall accuracy of these transformations and SDSS photometry is better than ~ 0.05 mag, as determined by direct comparison with non-SDSS observations obtained in the Johnson system (M. Fukugita, E. Grebel, J. Holtzman, unpublished).

2.2. The catalog of known asteroids

For positional matching of the moving objects observed by SDSS with cataloged asteroids we select the Asteroid Orbital Elements Database (ASTORB). ASTORB is a catalog of high-precision osculating orbital elements and other information on all numbered and a large number of unnumbered asteroids (but does not contain information on known numbered comets). The ASTORB catalog is distributed by the Lowell Observatory in the form of an ASCII file, containing single line records for each asteroid (Bowell 2001). The catalog is updated daily for addition of newly discovered objects, deletion of duplicate objects, and improvement of parameters of known objects. In this work we use the six osculating orbital elements, absolute magnitude and the phase-correction slope parameter which are necessary to predict the asteroid position and apparent magnitude at the time of SDSS scans. We also utilize the arc (in days) spanned by the observations used to compute the orbit to estimate the accuracy of orbital elements.

Orbital elements given in ASTORB are heliocentric and have been computed by a variable-timestep differential orbit correction algorithm, based on astrometric observations obtained from the Minor Planet Center. The perturbations from all the major planets, the Moon and the three largest asteroids (Ceres, Pallas and Vesta) have been taken into account. Absolute magnitude corresponding to the Johnson V band (H , see I01) is listed as numbers rounded to two, one, or no decimal places, with the number of decimal places reflecting assumed reliability. However, for unnumbered asteroids H is given to two decimal places regardless of its real accuracy. The slope parameter G (see section 2.3.2 below) is given for asteroids for which it is known, and for others, which are the overwhelming majority of the sample, assumed to be 0.15 (dimensionless).

We note that besides ASTORB there are several other asteroid orbital elements databases (e.g. Minor Planet Center Orbit Database²¹, Asteroid Dynamics Site²²). However, because of its widespread use and significant additional information supplied for each asteroid, we have

chosen to use ASTORB as a referent catalog for this study. The ASTORB version used here is from September 18, 2001, and contains 141,110 objects (29,074 of which are numbered asteroids).

2.3. Asteroid identification pipeline

The asteroid identification pipeline consists of three parts:

1. Propagation of the asteroid osculating orbital elements from ASTORB to the epoch of the SDSS observation
2. Computation of the asteroid positions and apparent magnitudes at the time of SDSS scan
3. Positional matching of SDSS moving objects with the known asteroids

2.3.1. Propagation of orbits

The ASTORB catalog contains osculating orbital elements computed for epoch near the current epoch, where “current” corresponds to the publishing date of the ASTORB catalog file. The orbital elements are propagated to the epoch of observation using the PROELE routine of OrbFit v1.8 *propag* library (Milani *et al.* 1999). We use the default dynamical model supplied with the OrbFit package. It includes gravitational perturbations of the Sun, Moon and the major planets and relativistic corrections, when necessary. The positions and masses for major planets are derived from JPL ephemeris DE405 (Standish 1998). Although the OrbFit package offers multiple choices for the numerical integration schemes, we use its automated selection procedures, which has been proven to be sufficiently accurate in practice (Jurić & Korlević 2000). The end result of the orbit propagation is a catalog of osculating orbital elements at the time of SDSS observation, which are then used for quick two-body (Sun and asteroid) computations of the ephemeris.

2.3.2. Computation of asteroid positions and apparent magnitudes

The calculations of positions are performed using a modified PREOBS routine from the same OrbFit package (the modification consists of enabling a two-body approximation). When calculating the positions of cataloged asteroids, they must satisfy the condition given by eq. 1. We solve the problem iteratively, until the difference between the positions calculated in two successive iterations is smaller than 0.001 arcsec. In practice the computations converge rapidly and usually reach the required accuracy with two to three iterations.

The apparent magnitudes are computed by OrbFit’s APPMAG subroutine from the cataloged values of the absolute magnitude H and the phase-correction slope parameter G . The phase-corrected apparent visual “catalog” magnitude, V_c , is obtained from (Bowell 1999)

$$V_c(\alpha) = H + 5 \log(R\Delta) - 2.5 \log((1 - G)\Phi_1(\alpha) + G\Phi_2(\alpha)) \quad (4)$$

²⁰Although SDSS has already produced more multi-color photometric measurements of asteroids than is available in the Johnson system, we use the Johnson system to ease comparison with earlier studies.

²¹Available from <ftp://cfa-ftp.harvard.edu/pub/MPCORB>

²²Available from <http://hamilton.dm.unipi.it/cgi-bin/astdys/astibo>

where R is the heliocentric, and Δ is the geocentric distance (expressed in AU), α is the “solar” angle (the angle between the Sun and the Earth as viewed from the asteroid, see I01), and Φ_1 and Φ_2 are the “phase-correction” functions. The latter are obtained from standard approximations

$$\Phi_i = \exp \left[-A_i \left(\tan\left(\frac{\alpha}{2}\right) \right)^{B_i} \right]; \quad i = 1, 2 \quad (5)$$

$$\begin{aligned} A_1 &= 3.33 & A_2 &= 1.87 \\ B_1 &= 0.63 & B_2 &= 1.22 \end{aligned} \quad (6)$$

Note that these approximations are formally different from the one used by I01 (eq. 11). However, they are similar numerically; the difference is less than 0.05 mag for $\alpha < 2^\circ$ and $\alpha \sim 6^\circ$, with the maximum value of ~ 0.15 mag at $\alpha \sim 3^\circ$.

2.4. Matching algorithm

The matching algorithm compares the list of predicted positions for the known asteroids with the list of SDSS objects flagged as moving by the photometric pipeline. After finding the nearest-neighbor pairs from the two lists, it requires that the distance between the two positions is less than 30 arcsec. The final sample is very insensitive to the precise value of this cut because for the majority (86%) of matched objects the distance between the two positions is less than 3 arcsec (centered on zero in both coordinates). The probability that a randomly chosen position would fall within 3 arcsec from a moving SDSS object is of the order 10^{-4} in the regions with the highest asteroid number density (64 deg $^{-2}$, I01), implying that only about one of the matches within 3 arcsec is a random association. The associated objects are taken to be positive positional identifications of an SDSS moving object with a cataloged asteroid.

3. THE MATCHING RESULTS

The SDSS photometric pipeline identified 12,668 moving objects selected as described by I01. There are 2801 cataloged asteroids whose computed positions are within the boundaries of the SDSS scans considered here, and 1633 have within 30 arcsec an object flagged by SDSS photometric pipeline as moving. Of these, only 1325 are unique objects because some asteroids were observed by SDSS more than once (I01). Detailed statistics for each run are listed in Table 1.

This comparison, taken at face value, implies that the SDSS sample is 58% complete. This is a much lower completeness than claimed by I01 (98%). We have visually inspected SDSS images around the positions of all 1168 ASTORB objects not found in the SDSS catalog (moving objects are easily recognized on 1x1 arcmin g-r-i color composites thanks to their peculiar appearances, see I01). There are 219 objects which are clearly moving, but were not recognized as such by the photometric pipeline. The most common reason for missing them are complex environments (bright stars with diffraction spikes, compact galaxy clusters, meteor and airplane trails, etc.) which are hard to “deblend” into individual sources. The revised, true completeness of the SDSS sample is thus 88%,

somewhat lower than 98% determined by I01. We note that I01 determined completeness using only data from run 756. For this run the completeness determined here is 91%. Furthermore, for all runs except for run 94 the completeness is higher than 90%. The overall completeness is below 90% only for run 94 (81%), which is one of the earliest SDSS commissioning runs, obtained while the telescope still did not have proper optics.

For the remaining 949 (34% of the total) ASTORB objects there is no visible SDSS source within 30 arcsec from the predicted position down to the sensitivity limit of $r^* \sim 22.5$. The most plausible explanation for these “missing” sources is inaccurate orbital elements (that is, their true position during SDSS observations is further than 30 arcsec from the predicted position). Indeed, the inspection of their entries in the ASTORB catalog indicates that the majority are either faint ($H \gtrsim 17$), have a small observational arc used to determine the orbital elements, or both.

3.1. The selection of asteroids with reliable orbital elements

SDSS observations demonstrate that a large fraction of known asteroids listed in the ASTORB catalog do not have sufficiently accurate orbital elements for reliable identification. The observational arc used to determine the orbital elements can be utilized as a rough estimate of their quality – the longer the arc spanned by the observations used to compute the elements, the more accurate are the positions. The ASTORB catalog also provides a more quantitative estimate for the accuracy of the orbital elements – the *Current ephemeris uncertainty (CEU)* parameter (1- σ absolute positional uncertainty), for an epoch near the catalog publishing date. However, the *CEU* cannot be propagated to the time of observation without the knowledge of covariance matrices for the orbital elements solution (cf. Muinonen & Bowell 1993), and we use arc to make a quality cut.

The relationship between the *CEU* and the arc is shown in Figure 1. The top panel shows each object from the ASTORB catalog as a dot, and the bottom panel shows the arc histograms for all objects by a full line, and separately for objects with *CEU* > 30 arcsec as a shaded region. As evident, the requirement that arc > 300 days successfully eliminates most of the asteroids with large CEUs, and selects 1634 objects (out of 2801). Note that the resulting sample is insensitive to the precise value of this cutoff because of the non-continuous distribution of data. A further constraint is based on the predicted apparent magnitude. We only select asteroids that are reliably detectable by SDSS; they satisfy $14 \leq V_c \lesssim 21.5$ (I01). This cut has only a minor importance and selects 1612 objects (relaxing the limit to 22.5 adds 3 objects).

The absolute magnitudes for selected (hereafter “reliable”) and removed asteroids (hereafter “unreliable”) are compared in Figure 2. The solid line shows the absolute magnitude distribution for all 2801 asteroids, and the dashed and dotted lines show the distributions for the 1612 (58%) reliable and 1189 (42%) unreliable asteroids, respectively. It is evident that the asteroids with unreliable orbital parameters are mostly faint, and dominate the sample for $H > 15.5$. The subsample of reliable asteroids appears complete for $H < 14$.

The removal of the sources with unreliable orbital elements is efficient, but not perfect. The matching statistics for the subsample of ASTORB asteroids with reliable orbits is listed in Table 2. Out of 1612 objects, 1335 are matched within 30 arcsec to objects automatically recognized by the photometric pipeline as moving. Of the remaining 277 objects, 173 are visually identified as moving, and the remaining 104 objects do not have an SDSS moving source within 30 arcsec. For each of these 104 objects, the predicted position has been independently verified using the on-line AstDys orbital calculator²³. In summary, the arc > 300 days cut decreases the fraction of ASTORB catalog asteroids with unreliable orbital elements from 34% to 6%. The SDSS completeness based on the reduced sample is 89% (=1335/1508), consistent with the estimate based on the full sample (88%). Since the removed objects are predominantly faint, this agreement indicates that the SDSS completeness is not strongly dependent on apparent magnitude.

3.2. The Matching Statistics

The sample of 1335 matched objects that were detected by the photometric pipeline, together with the sample of 173 visually confirmed objects from the ASTORB catalog, can be used to determine whether the completeness of the SDSS automatically detected sample depends on magnitude. The top panel in Figure 3 compares the absolute magnitude distribution, as listed in the ASTORB catalog, for the 1335 matched objects (dashed line) to the distribution of all 1612 objects (full line). The bottom panel shows the fraction of matched sources in each magnitude bin, shown by full line with error bars. The horizontal line is added to guide the eye and represents the overall completeness of 89%. As evident, there is no significant correlation between the fraction of matched sources and the magnitude.

We have also tested for correlations between the fraction of matched sources and the phase angle, the object's color and apparent magnitude and did not find any significant dependences.

3.2.1. The apparent magnitude distribution

The top panel in Figure 4 compares the apparent magnitude distribution for asteroids from the ASTORB catalog that are within the boundaries of SDSS scans discussed here (full line), with the apparent magnitude distributions for moving objects detected by SDSS. The distribution for all SDSS objects is shown by the dashed line and the distribution for matched objects by the dotted line. We use the calculated V_c magnitudes for the ASTORB asteroids and synthesized V_o magnitudes for SDSS asteroids (including the matched sample; this is why the dotted line is above the solid line for $V \gtrsim 19$). At first sight it appears that the SDSS sample is significantly incomplete. The bottom panel shows the number ratio of ASTORB and SDSS asteroids in each magnitude bin, and indicates that the SDSS completeness is as low as 60% for $V \lesssim 17.5$. However, as demonstrated in the previous section (see Figure 3), the SDSS catalog includes 89% of ASTORB asteroids, and the mismatch in Figure 4 is simply due to an offset in apparent magnitude scales.

The existence of such an offset is further corroborated by the direct comparison of the calculated (V_c , based on the ASTORB catalog) and synthesized (V_o , based on SDSS observations) magnitudes for the 1335 matched objects. The top panel in Figure 5 shows the histogram of the difference $V_o - V_c$ for all 1335 objects by the full line. The median offset is 0.41 mag (the SDSS values are fainter), with the root-mean-scatter of 0.35 mag. The dotted and dashed lines show the $V_o - V_c$ histograms for the 400 blue asteroids and the 935 red asteroids selected by their a^* color as described in I01. It is evident that the apparent magnitude offset depends on the asteroid color; the median offsets are 0.34 and 0.44 for the blue and red subsample, respectively. This dependence is further illustrated in the bottom panel, where the $V_o - V_c$ difference is plotted as a function of the asteroid color a^* . The $V_o - V_c$ histograms shown in Figure 5 are not symmetric and give a hint of two components: one centered at ~ 0.1 is independent of color, and the other one centered at ~ 0.4 - 0.5 that appears to be color-dependent. This bimodality is more pronounced for $V \lesssim 17$, as discernible in Figure 6.

It is of obvious interest to find out whether the offset between predicted and observed magnitudes is particular to the ASTORB catalog, or perhaps also present in other available databases. For this test we chose the Minor Planet Center Orbit Database from September 5, 2001. It contains similar data to the ASTORB catalog for 115,797 objects; 115,583 of these objects are common to both catalogs. The top panel in Figure 7 compares the listed absolute magnitudes for the objects in common. As evident, the catalogs contain two “populations” of object. For about half of the sample the difference between the two values is less than 0.1 mag, while for the remaining 57,719 objects it is larger than 0.1. The median value for the latter is ~ 0.4 . We did not find any correlation between the difference in absolute magnitude and other cataloged quantities. In particular, there is no correlation between this difference and the mean absolute magnitude. Given only two catalogs, it is not possible to tell which one is responsible for the offset in magnitude scale. Fortunately, the catalogs can be compared to SDSS observations of the matched objects for which the photometric errors are not larger than 0.05 mag. The bottom panel in Figure 7 compares the $V_o - V_c$ histograms obtained with the data from the ASTORB catalog (full line) and from the MPC catalog (dashed line). As evident, the median discrepancy between the cataloged magnitudes and the magnitudes measured by SDSS is about twice as large for the ASTORB catalog as for the MPC catalog (~ 0.4 mag vs. ~ 0.2 mag). Based on this finding alone, it would seem that the MPC catalog should be used for the matching purposes. However, the MPC catalog contains fewer objects than the ASTORB catalog; out of the 1633 matched objects only 1387 are found in the MPC catalog (implying an upper limit on the completeness of the MPC catalog of 85%).

The discrepancy in the absolute magnitude scale significantly affects the number of objects brighter than a given limit. Assuming that the number counts follow a $\log(N) = C + 0.6 V$ relation (I01), the MPC catalog will imply 1.32 more objects than SDSS measurements, and the ASTORB catalog will imply 1.74 more objects than

²³Available at <http://hamilton.dm.unipi.it/cgi-bin/astdys/astibo>

SDSS measurements.

Although the uncertainty in the values of V_o synthesized from the SDSS photometry is probably smaller than 0.05 mag, it is prudent to verify the offset in magnitude scales using non-SDSS observations. Krisciunas, Margon and Szkody (1998) obtained photometric measurements in SDSS-like bands for 15 bright asteroids. Table 3 lists their measurements, and the synthesized, V_o , and predicted, V_c , magnitudes. The $V_o - V_c$ difference is shown as a function of V_o in Figure 8. Asteroids with H in the ASTORB catalog given to two decimal places are marked as circles, and those with one decimal place are marked by triangles. The former are predominantly at the bright end where they scatter around zero offset, indicating that the photometric transformations from SDSS to Johnson V band are not biased. The latter are found at the faint end and indicate a similar offset in the magnitude scales as discussed above.

The correlation between the magnitude offset and the asteroid color indicates that the discrepancy is probably due to differences in the standard Johnson V magnitude and the “visual” magnitude listed in the ASTORB catalog. A most likely cause is the fact that the majority of current asteroid surveys (which produced most of the entries in the catalogs) are obtained with open, unfiltered CCDs since they are mostly concentrated on astrometric measurements. Thus, the measured magnitudes are not equivalent to the standard Johnson V band, and furthermore, the discrepancy depends on the asteroid color. Indeed, as discussed by Jedicke, Larsen & Spahr (2002), the reporting of observed asteroid magnitudes is notoriously inconsistent. For example, the Minor Planet Center regularly weights magnitudes reported by different groups according to their historical photometric accuracy and transforms them to the V band, while the ASTORB database uses the unweighted photometry in observed bands in determining the absolute magnitude.

Correcting the predicted magnitudes for a median offset of 0.41 mag shifts the V_c histogram for ASTORB asteroids shown in Figure 4 by the right amount to make the matched fraction consistent with the estimate shown in Figure 3. We show the shifted histogram in Figure 9.

4. THE COLOR SEGREGATION IN THE ASTEROID BELT

As discussed in Section 2.1, we joined the 1335 matched objects from the SDSS Early Data Release with 1306 matched objects from unreleased runs, to form the final sample of 2641 unique matched objects. This is the sample used in our analysis of the color segregation in the asteroid belt.

4.1. Asteroid dynamical families

Asteroid dynamical families are groupings of asteroids in proper²⁴ orbital elements space, widely thought to be the results of collisional disruptions of parent bodies (larger asteroids). Pioneered by Hirayama (1918), the number and understanding of asteroid families has increased dramatically thanks to a number of factors – a large increase in the number of discovered asteroids, more advanced knowledge of celestial mechanics and more sophisticated cluster-

discovery techniques. For a detailed review see Chapman *et al.* 1989.

In a recent study, Zappala *et al.* (1995) identified 32 families and 31 “clumps” using a hierarchical clustering method in a sample of 12,487 asteroids. However, even a rough taxonomic classification exists only for a small number of these objects. The knowledge of the chemical composition of each family could provide further clues about the physical properties of the parent body. SDSS photometric data can remedy this problem because the asteroids segregate in SDSS multi-dimensional color space. In this section we analyze the correlation between SDSS colors and the distribution in orbital parameter space for 2641 matched asteroids.

In order to study the correlation between SDSS colors and the various taxonomic systems, we have cross-referenced the 1335 matched asteroids from the SDSS EDR sample with a catalog of asteroids with known taxonomy obtained at the Planetary Data System Small Bodies Node. The catalog contains information on 1199 objects, with Tholen (978 objects), Barucci (438 objects), Tedesco (357 objects), Howell (112 objects) and Xu (221 objects) taxonomies. There are only seven asteroids with taxonomical information present in the SDSS sample, and are listed in Table 4. This small number is not surprising given that the SDSS saturation limit ($V \sim 14$) is usually below the faint limit for most taxonomy surveys. The seven asteroids have SDSS colors in agreement with their taxonomic types (I01).

4.2. The Color properties of Asteroid Families

The 2641 known asteroids observed by SDSS have been cross-referenced with the catalog of proper orbital elements produced by Milani *et al.* (1999) and distributed through AstDyS. The resulting sample includes 1768 asteroids with available proper elements. Hereafter we separate this sample into 531 blue asteroids ($a^* < 0$, see I01), and 1237 red asteroids ($a^* > 0$). As discussed by I01, there is a possibility that red asteroids with $i^* - z^* < -0.25$ form a separate group in color-color space, and we treat separately the 131 asteroids with such colors. In the remainder of this Section, we analyze the differences in the distributions of these subsamples in the three-dimensional space spanned by the semi-major axis, a' , inclination, i' , and eccentricity, e' .

Figure 10 displays the distribution of asteroids in the a' vs. $\sin(i')$ plane. The top panel shows 67,917 asteroids from the ASTORB database which have known proper orbital elements (from Milani *et al.* 1999), marked as dots. The major dynamical families (also known as the Hirayama families) Eos, Koronis and Themis, are clearly visible at $a' > 2.8$ AU; their approximate $(a', \sin(i'))$ positions are (3.0, 0.18), (2.9, 0.03) and (3.15, 0.02), respectively. The 4:1, 3:1, 5:2 and 2:1 mean motion resonances with Jupiter at 2.065, 2.501, 2.825 and 3.278 AU (the latter three correspond to the Kirkwood gaps) are also evident. The ν_6 resonance is visible as a strong cutoff of asteroid density at high inclinations in the $2.065 < a' < 2.501$ region. Due to using a sample which is about five times

²⁴Proper orbital elements are nearly invariants of motion and thus are well suited for discovering objects with common dynamical history. They are different from the osculating orbital elements which describe the current motion. For more details see Milani & Knežević 1992, and references therein.

larger, the various families are more easily discerned than in Zappala *et al.* (1995); however, the distribution shown here agrees with their results. The remaining two panels show the same distribution as isodensity contours, with the matched asteroids shown by open circles. The middle panel shows blue asteroids ($a^* < 0$), and the bottom panel shows red asteroids ($a^* > 0$). A subset of red asteroids with $i^* - z^* < -0.25$ is shown by crosses; most are found around $a' \sim 2.2$ – 2.5 and $\sin(i') \sim 0.12$. Figures 11 and 12 are analogous to Figure 10, and show the asteroid distributions in the e' vs. a' and e' vs. $\sin(i')$ planes. Figure 12 shows the distributions in three belt regions selected by the positions of resonances.

For studying the chemical composition of various families, we adopt the family definitions by Zappala *et al.* (1995). The distribution of blue and red asteroids in the a' - $\sin(i')$ - e' space confirms the earlier results for the three largest families. Koronis and Eos family seem to be mostly composed of red asteroids: 40 out of 45 (89%) asteroids associated with the Koronis family are red, and 77 of 98 (79%) are red for the Eos family. On the other hand, the Themis family is predominantly blue: 41 out of 43 (95%) members have $a^* < 0$. All three major families are located in the outer belt. From other families in the outer belt we find two members of the Brasilia family, both are blue.

The middle portion of the belt, between the 3:1 and 5:2 resonances ($2.5 < a' < 2.8$) contains a number of smaller families. The SDSS sample discussed here provides good color information for five of them. The Maria family is found to be red with 16 out of 21 asteroids with $a^* > 0$. The more numerous Eunomia family also displays red characteristics (97 red and 7 blue members). On the other hand, the Adeona family and the Dora family are blue (12 out of 15, and all 11 members are blue, respectively). The Ceres family seems to be a mix of both asteroid types, with 11 blue and 24 red asteroids. Other smaller families have fewer identified members; we find marginal evidence that the Hoffmeister, Rafita and Hestia families are predominantly red, while the Misa and Taiyuan families are predominantly blue.

The inner part of the belt is dominated by the Vesta/Flora complex and at lower inclinations by the Nysa/Polana and Massalia families. The Nysa/Polana group is found to include a mix of red and blue asteroids, with red asteroids dominating the sample in approximately 3:1 ratio (51 red and 19 blue). This is the strongest mixing of blue and red asteroids found for the families studied here, and supports the claim by Cellino *et al.* (2001) that the Nysa/Polana group includes two independent families, one composed of S (red) asteroids and the other including low-albedo F-like (presumably blue) objects.

We note that for many families we find a substantial fraction ($\gtrsim 10\%$) of the minor component (e.g. $\sim 20\%$ of the red Eos family members are blue). It is not easy to determine whether this mixing is due to the background contamination (i.e. by asteroids that are not family members), or due to non-homogeneous structure of the parent bodies²⁵. A robust analysis necessarily involves precise definitions of families, and a careful study of the multi-dimensional color distribution for each family. Such an

analysis is beyond the scope of this work and will be presented in a future publication.

4.3. The V and J type Asteroids

I01 analyzed the distribution of 316 asteroids in SDSS color space by producing synthetic SDSS colors from the spectral measurements obtained by the SMASS project (Xu *et al.* 1995). In addition to the two major color types, there was an indication that red asteroids with extremely blue $i^* - z^*$ colors may form a separate class (see Figure 10 in I01). This notion was also supported by the taxonomic classification of asteroids with such colors; they were all classified as the J and V asteroids associated with the Vesta family (Binzel & Xu 1995). The proper orbital elements for 131 objects with $i^* - z^* < -0.25$ discussed here can be used to test whether these objects cluster in the a' - $\sin(i')$ - e' space.

The Vesta family occupies a very small volume in the orbital parameter space centered at $a' \sim 2.35$, $\sin(i') \sim 0.12$, and $e' \sim 0.1$. If the asteroids with extreme $i^* - z^*$ colors can be interpreted as J and V taxonomic types, then they should be concentrated in that region, rather than scattered throughout the belt as blue and red asteroids are. The 131 red asteroids with $i^* - z^* < -0.25$ are marked in Figures 10, 11 and 12 as crosses; it is evident that they are clustered around the Vesta family. This is better seen in Figure 13 where we show only the small region of the orbital parameter space that is relevant for the Vesta family ($2.1 < a' < 2.5$ and $0.09 < \sin(i') < 0.15$). The red asteroids are shown as circles and the subset of 99 objects with $i^* - z^* < -0.25$ as crosses. The large dot marks the position of Vesta. The box outlined by the solid line is the core region ($2.28 < a' < 2.4$ and $0.1 < \sin(i') < 0.13$), and the box outlined by the dashed line is the tail region ($2.4 < a' < 2.49$ and $0.1 < \sin(i') < 0.13$), as proposed by Zappala *et al.* (1995). Since a very high fraction of objects with $i^* - z^* < -0.25$ (99 out of 131 in the displayed region, and 50 and 29, respectively, in the outlined core and tail regions) is found in the region of the orbital parameter space that is associated with the Vesta family, we conclude that the $i^* - z^*$ color provides a reasonably reliable method for selecting objects from the Vesta family.

5. DISCUSSION

This work demonstrates the feasibility of significantly increasing the number of asteroids with both accurate multi-color photometry and known orbital elements by matching SDSS-detected asteroids with the catalogs of known asteroids. The sample discussed here includes ~ 2600 asteroids; when SDSS is completed the final catalog could be more than ten times larger. Such a large sample can be used for a compositional analysis of the asteroid families at a level of detail which was not previously possible. We show here that SDSS color information is sufficient to reveal strong color segregation among the asteroid families: most are dominated by either blue or red objects.

We showed that the SDSS photometric pipeline automatically flags $\sim 90\%$ of observed moving objects, with a contamination level of only a few percent. This is a better performance than for any other similar code. The directly

²⁵It is also possible that there is an asteroid color type that occupies the region around the boundary between blue and red types; e.g. see Figure 10 in I01

determined completeness is lower than the 98% claimed by I01, though without any significant consequences for their results.

It is somewhat surprising that only 66% of the known asteroids listed in the ASTORB catalog have sufficiently accurate orbital elements to predict their positions to better than 30 arcsec. However, most objects with unreliable orbital elements can be efficiently removed. It is much more perplexing that there is a large offset in magnitude scales, not only with respect to SDSS, but also among the standard asteroid databases. This offset results in an overestimate of the number of asteroids brighter than a given magnitude limit by factor 1.7, and resolves the discrepancy between the SDSS asteroid count normalization and the number of asteroids listed in ASTORB catalog.

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TABLE 1
ALL ASTORB ASTEROIDS MATCHED TO SDSS OBJECTS

Run	ASTORB asteroids	SDSS objects	Matched objects	Missed by <i>photo</i>	Not found ASTORB
94	663	2757	395	94	174
125	648	2844	426	48	174
752	712	3422	373	33	306
756	778	3645	439	44	295
All	2801	12668	1633	219	949
Unique	2170		1325		

TABLE 2
RELIABLE ASTORB ASTEROIDS MATCHED TO SDSS OBJECTS

Run	ASTORB asteroids	SDSS objects	Matched objects	Missed by <i>photo</i>	Not found ASTORB
94	391	2757	290	82	19
125	402	2844	350	32	20
752	374	3422	318	25	31
756	445	3645	377	34	34
All	1612	12668	1335	173	104
Unique	1239		960		

TABLE 3
THE PHOTOMETRIC OBSERVATIONS OF ASTEROIDS^a

No.	Name	Type	1997 Date	UT	r*	g*−r*	V _{mea} ^b	V _{cat} ^c	V _{mea} −V _{cat}
446	Aeternitas	A	30 Sep	11:09	14.33	0.61	14.58	13.43	1.14
702	Alauda	C	30 Sep	11:47	12.75	0.45	12.93	12.90	0.03
82	Alkmene	S	28 Sep	10:57	12.43	0.64	12.69	12.72	-0.02
774	Armor	S	28 Sep	03:52	12.97	0.71	13.26	12.94	0.32
371	Bohemia	AS	4 Oct	05:05	12.64	0.65	12.90	12.83	0.07
349	Dembowska	R	29 Sep	11:54	10.28	0.70	10.57	10.57	-0.00
433	Eros ^b	S	30 Sep	12:26	13.30	0.67	13.57	13.09	0.48
480	Hansa	S	29 Sep	09:59	12.25	0.63	12.51	12.61	-0.10
10	Hygeia	C	28 Sep	12:05	11.43	0.46	11.61	11.54	0.06
683	Lanzia	C	1 Oct	03:35	14.23	0.48	14.42	13.78	0.64
68	Leto	S	27 Sep	10:37	10.11	0.66	10.38	10.53	-0.15
149	Medusa	S	29 Sep	06:12	12.57	0.68	12.85	12.96	-0.11
196	Philomela	S	28 Sep	08:51	11.41	0.64	11.67	11.56	0.10
314	Rosalia	C	5 Oct	03:15	13.95	0.46	14.13	13.81	0.32
138	Tolosa	S	5 Oct	04:11	11.57	0.69	11.85	11.92	-0.07

^aObservations with SDSS-like filters by Krisciunas, Margon & Szkody (1998).

^bThe synthesized Johnson V magnitude based on observations using SDSS-like filters.

^cThe predicted catalog-based Johnson V magnitude.

TABLE 4
 ASTEROIDS WITH KNOWN TAXONOMY IN THE SDSS
 EDR SAMPLE

Asteroid	Taxonomy ^a	a^*	$i^* - z^*$
(220) Stephania	XC (Tholen)	-0.062	-0.012
(1628) Strobil	X (SMASS)	0.024	0.079
(1679) Nevanlinna	X (SMASS)	-0.048	0.036
(1711) Sandrine	S (Tholen)	0.074	0.042
(1772) Gagarin	S (SMASS)	0.135	0.003
(2215) Sichuan	S (SMASS)	0.097	0.011
(5118) Elnapoul	S (SMASS)	0.151	-0.022

^aThe asteroid taxonomic classification and its source.

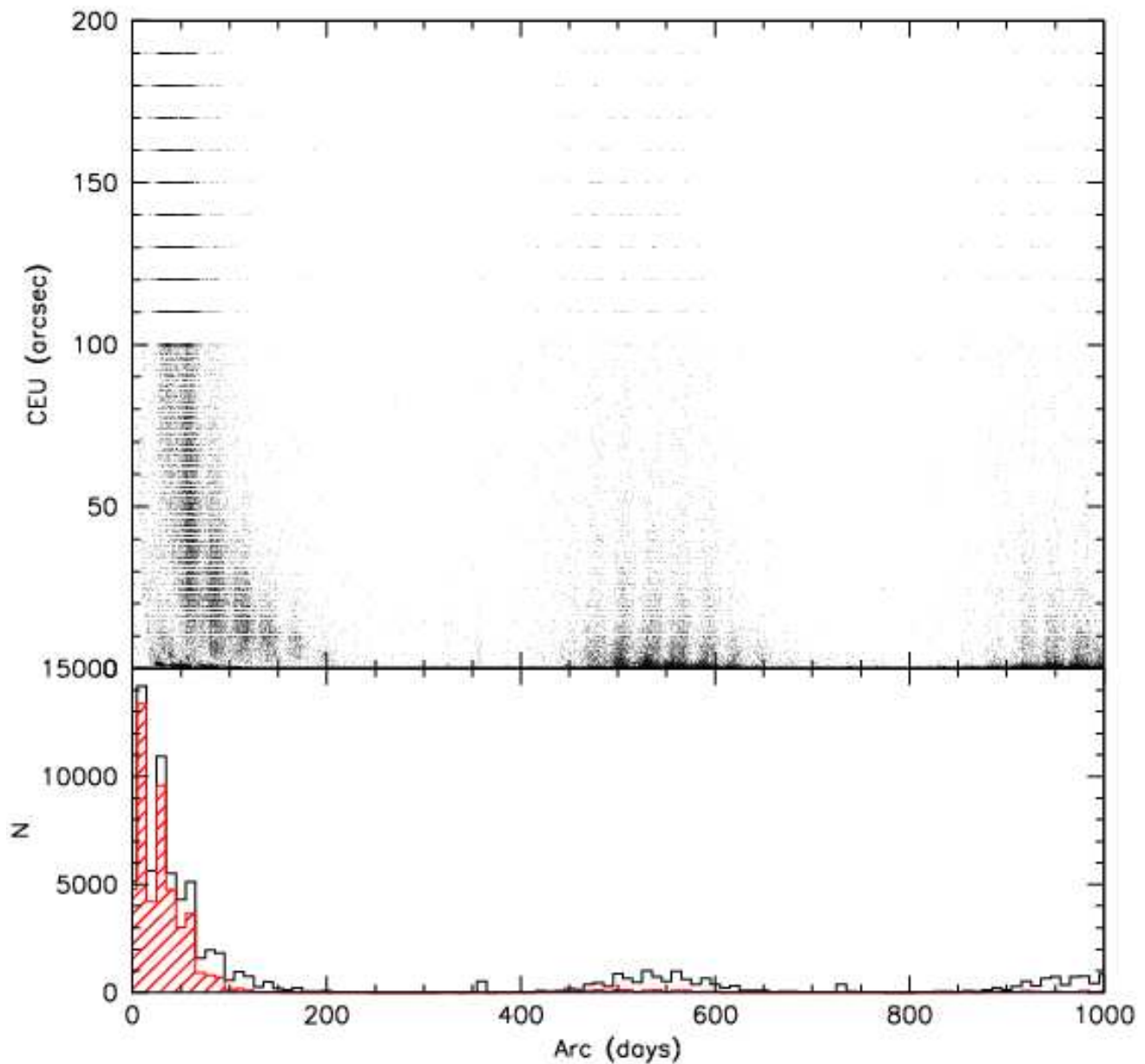


FIG. 1.— The relation between the observational arc and the current ephemeris uncertainty (CEU) for the 141,110 objects from the ASTORB catalog, marked by dots. The discretization is caused by truncation. The bottom panel shows the arc histograms for all objects, and separately for objects with $CEU > 30$ arcsec, shown by the shaded area. As evident, most of the latter are concentrated in the first peak and are eliminated by the condition $\text{arc} > 300$ days.

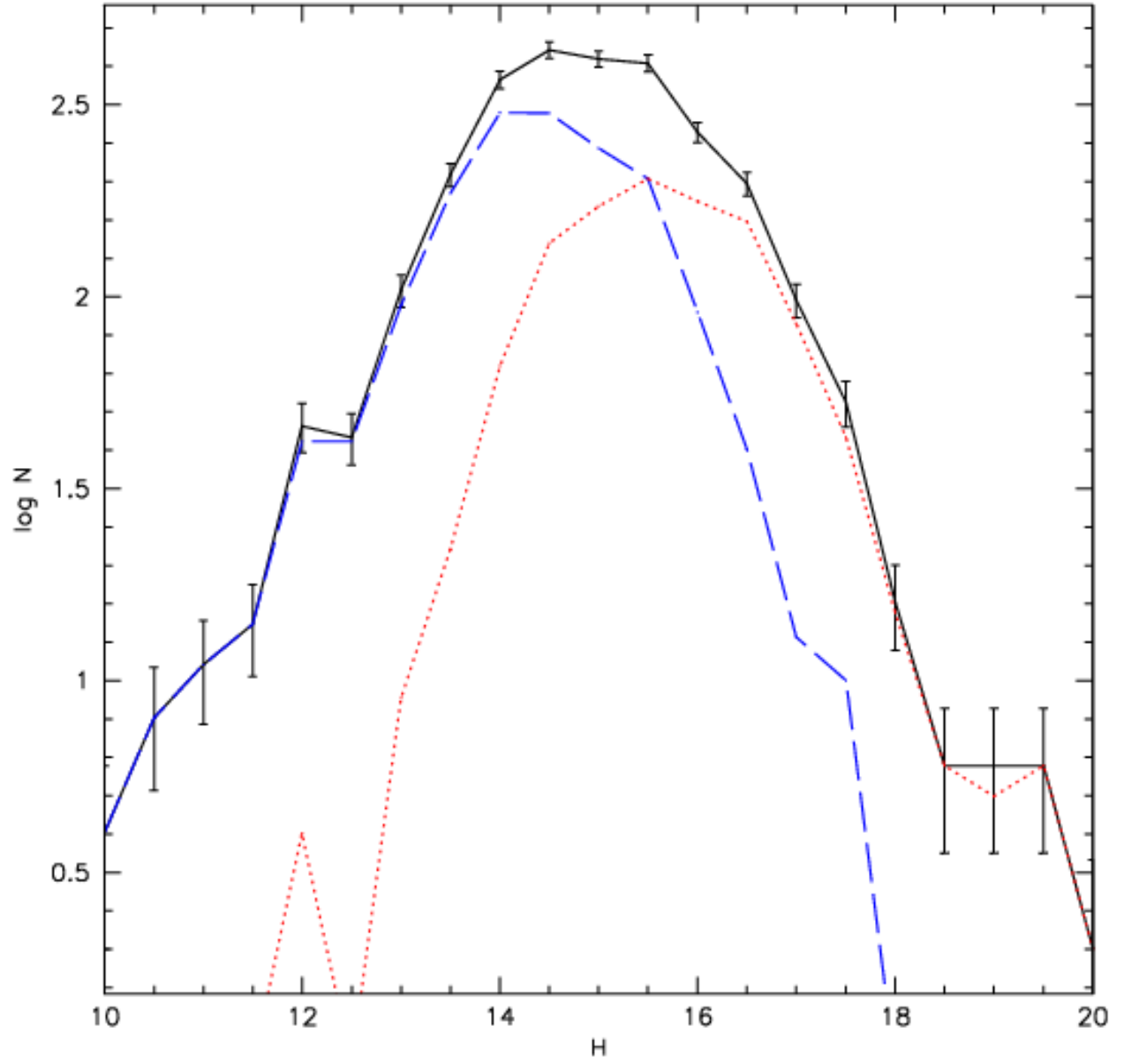


FIG. 2.— The solid line shows the absolute magnitude distribution for the 2801 known asteroids from the ASTORB database which are expected to be observed in SDSS scans discussed here. The distribution for the 1612 asteroids with most reliable orbital elements is shown by the dashed line, and the dotted line show the distribution for the remaining sources with less reliable orbital elements.

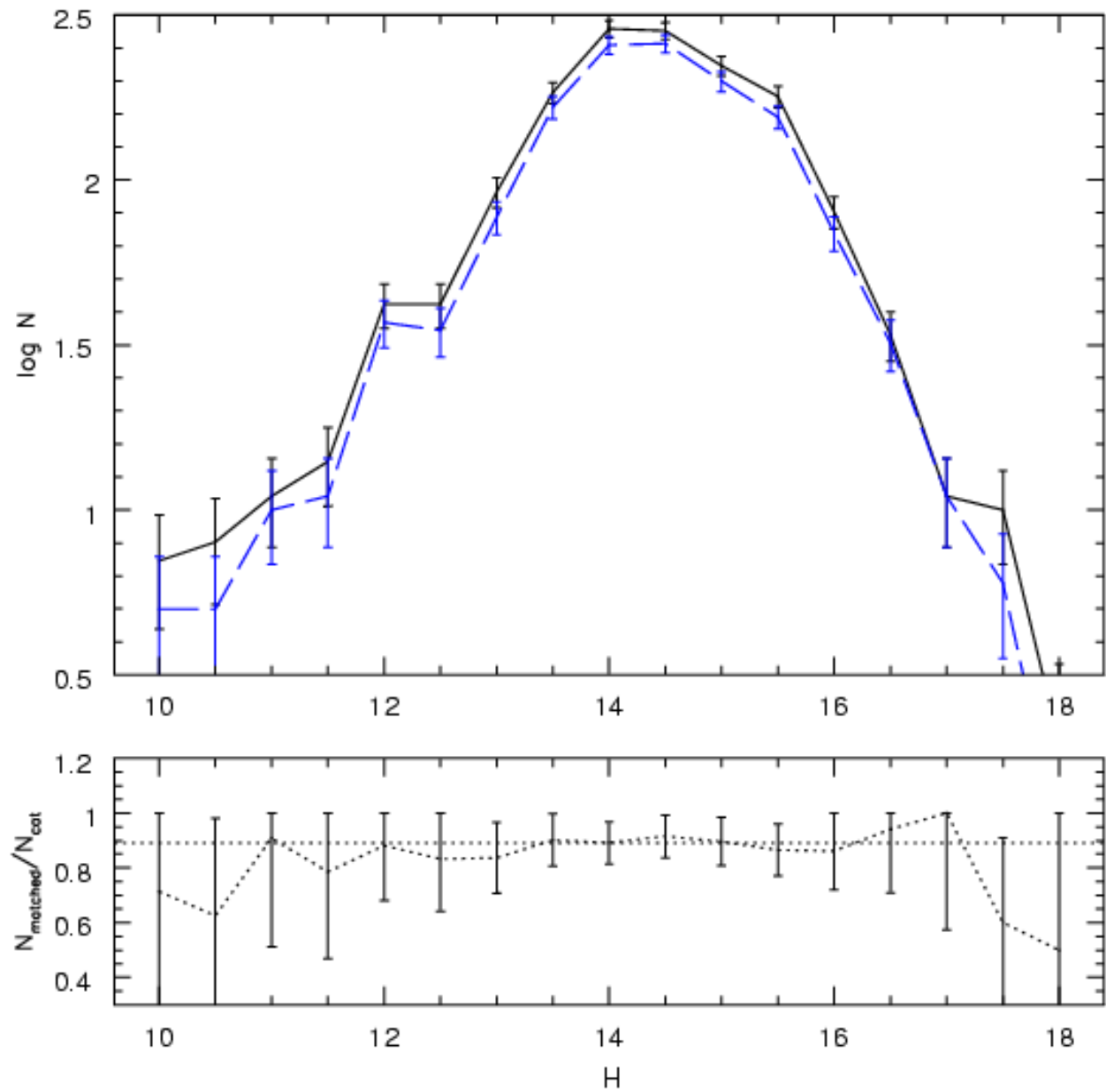


FIG. 3.— The top panel shows the absolute magnitude distribution, as listed in the ASTORB catalog, for the 1335 objects recognized as moving by the SDSS photometric pipeline and matched within 30 arcsec to an object from the ASTORB catalog (dashed line), and the distribution of all 1508 moving objects selected from the ASTORB catalog (full line). The bottom panel shows the fraction of matched sources in each magnitude bin, shown by full line with error bars. The horizontal line is added to guide the eye and represents the overall SDSS completeness of 89%. Note that there is no significant correlation between the fraction of matched sources and the magnitude.

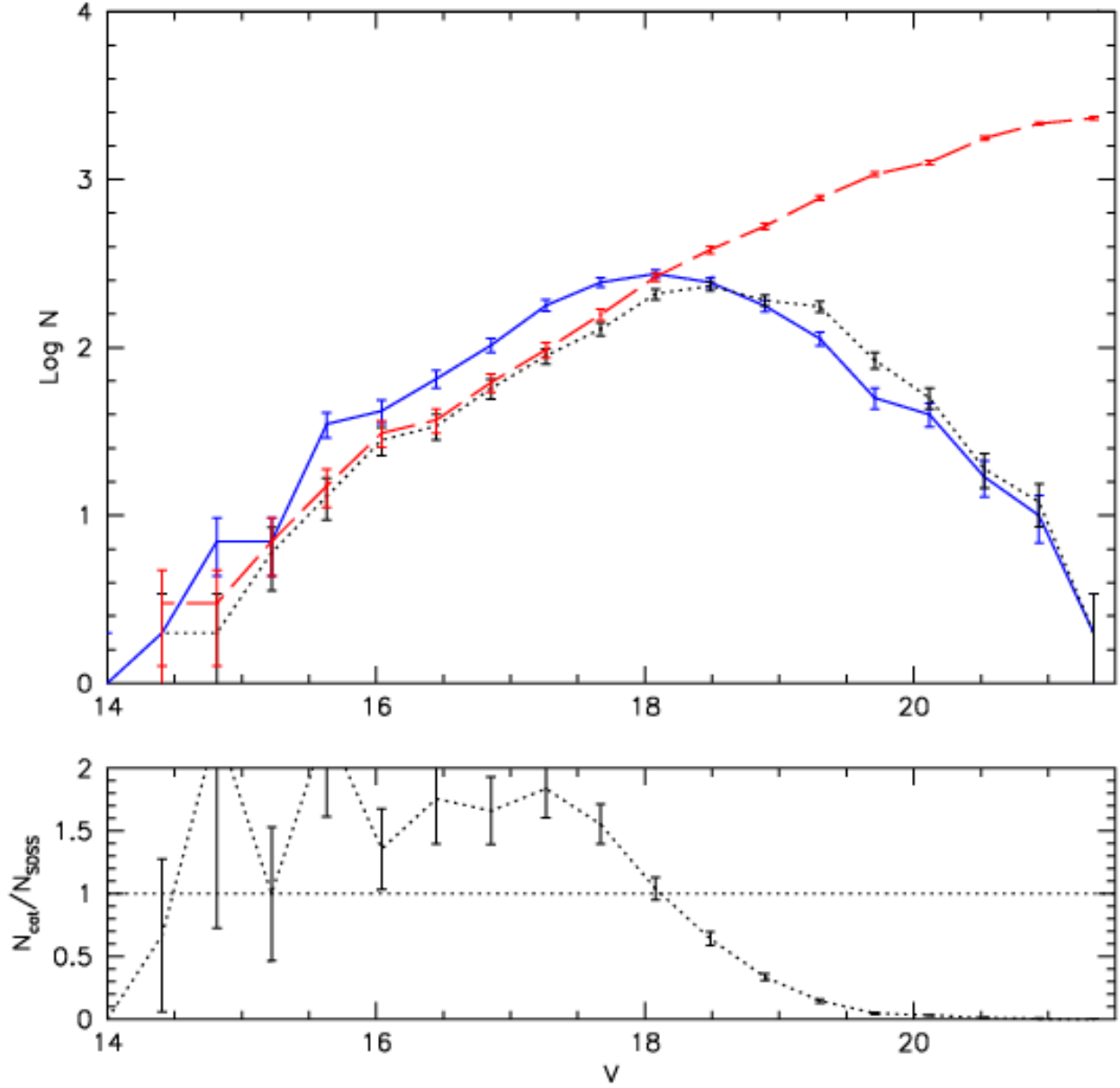


FIG. 4.— The top panel compares the apparent magnitude distribution for asteroids from the ASTORB catalog that are within the boundaries of SDSS scans discussed here (solid line), with the apparent magnitude distributions for moving objects detected by SDSS. The distribution for all SDSS objects is shown by the dashed line and the distribution for matched objects by the dotted line. The bottom panel shows the number ratio of ASTORB and SDSS asteroids in each magnitude bin. Although it appears that the SDSS completeness is as low as 60% for $V \lesssim 17.5$, the SDSS catalog includes 89% of ASTORB asteroids, and the mismatch is due to an offset in the apparent magnitude scales.

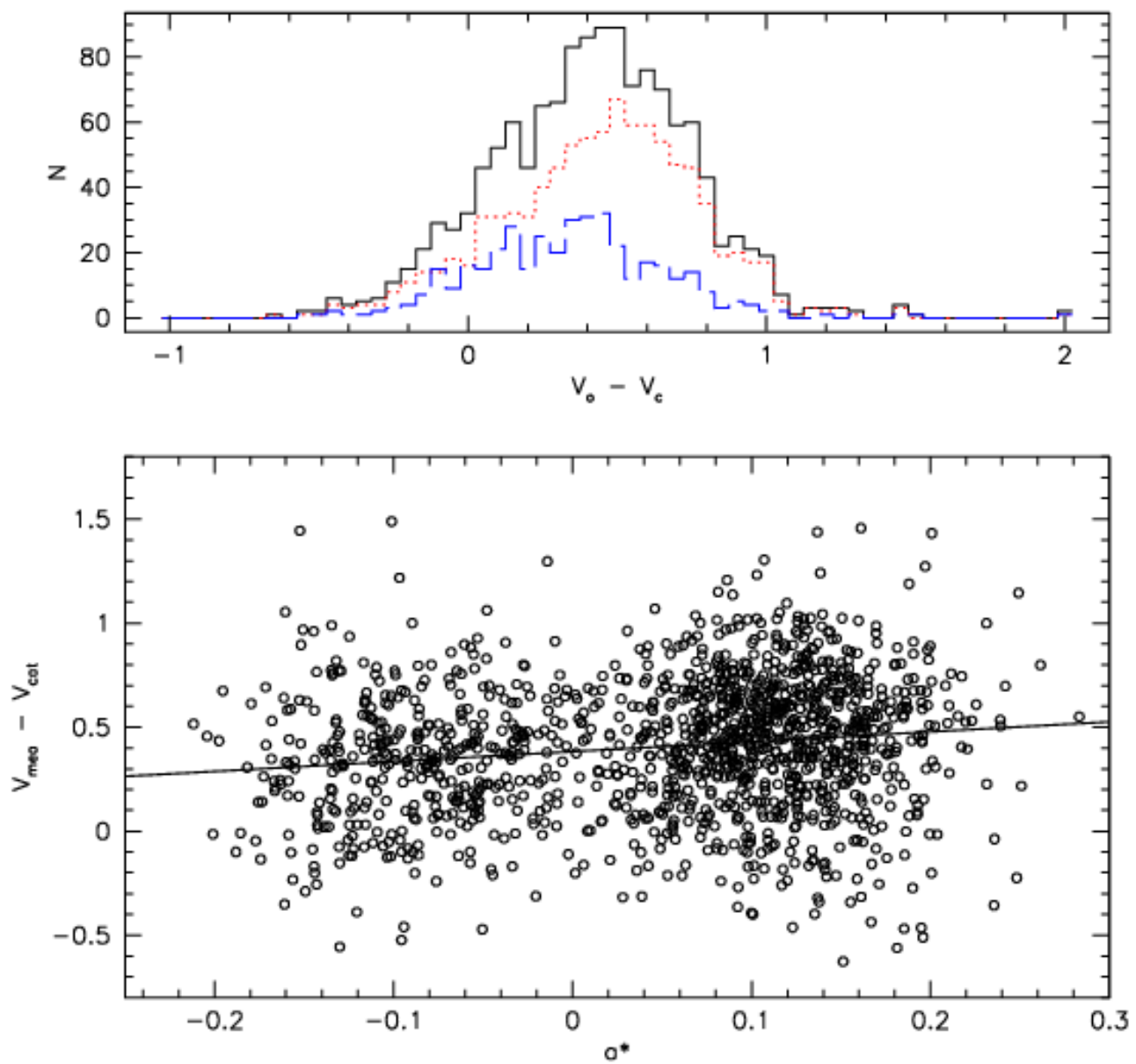


FIG. 5.— The $V_o - V_c$ histogram for all 1335 objects is shown in the top panel by the full line. The dotted and dashed lines show the analogous histograms for the 400 blue asteroids and the 935 red asteroids. The bottom panel shows the $V_o - V_c$ difference as a function of the asteroid color a^* , and a best linear fit.

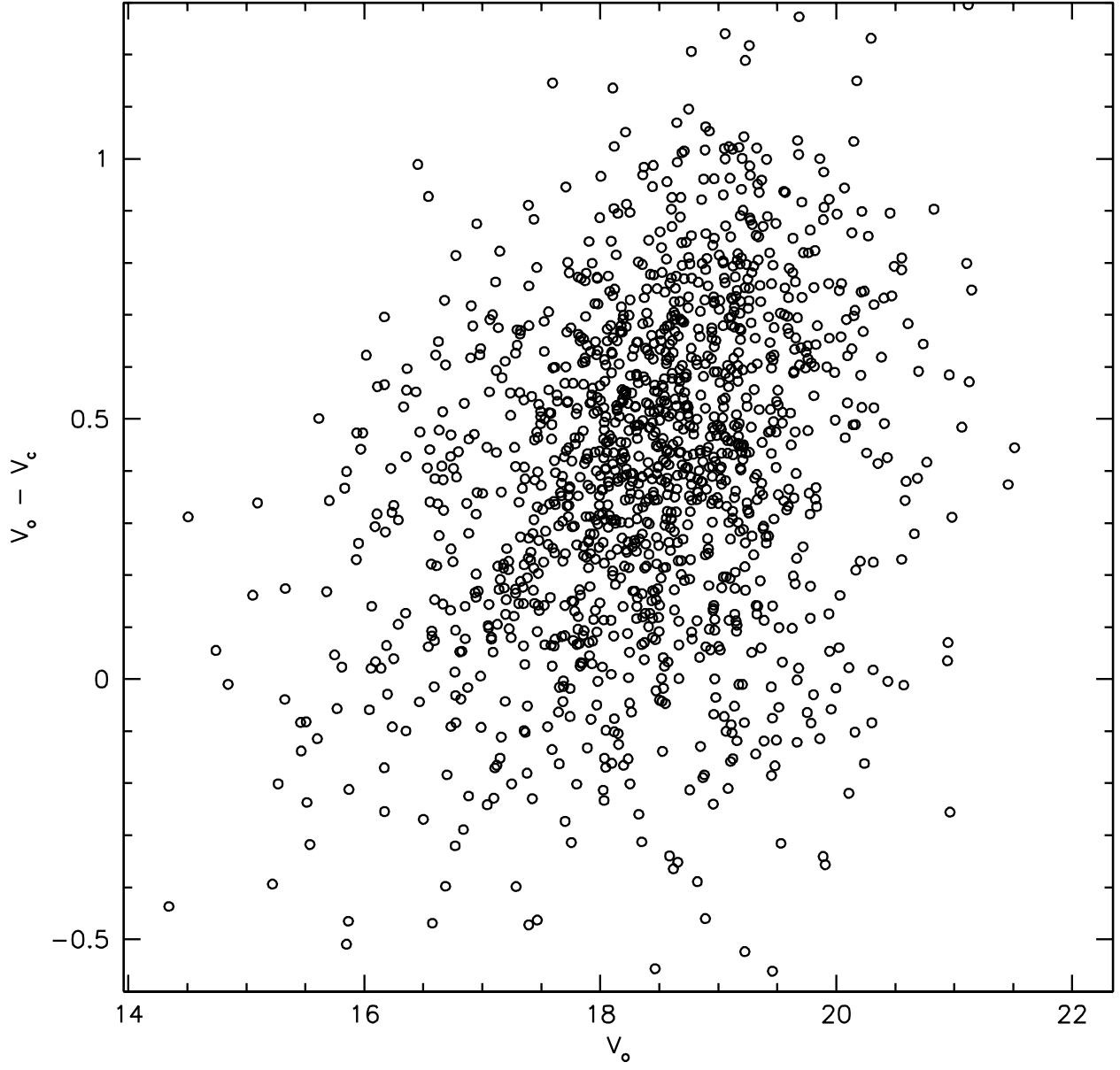


FIG. 6.— The $V_o - V_c$ difference for all 1335 objects as a function of V_o .

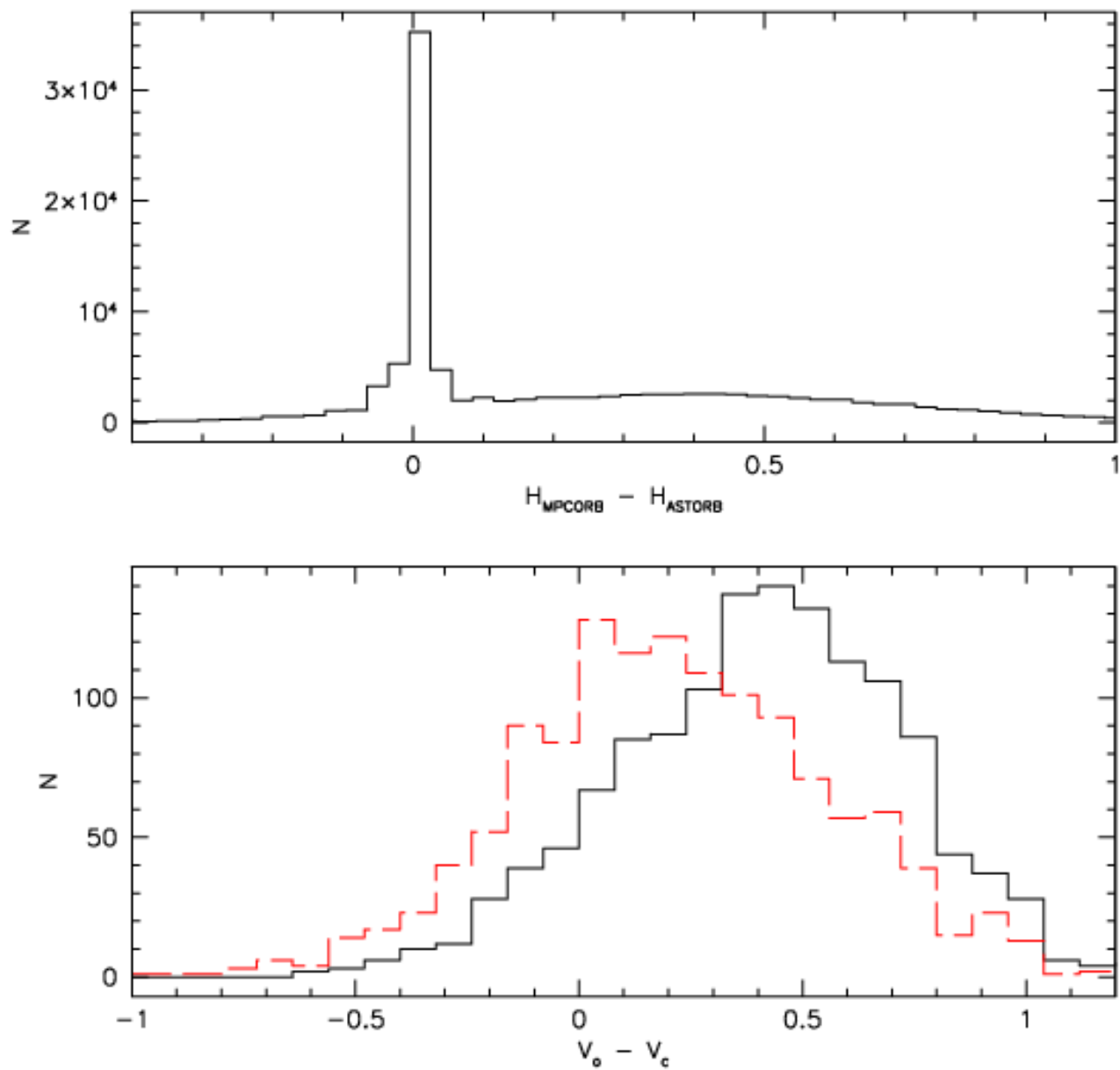


FIG. 7.— The top panel compares the difference in absolute magnitudes for the 115,583 common objects listed in the MPC and ASTORB catalogs. The bottom panel compares the difference between the magnitude observed by SDSS and predicted magnitudes for the ASTORB catalog (full line), and for the MPC catalog (dashed line).

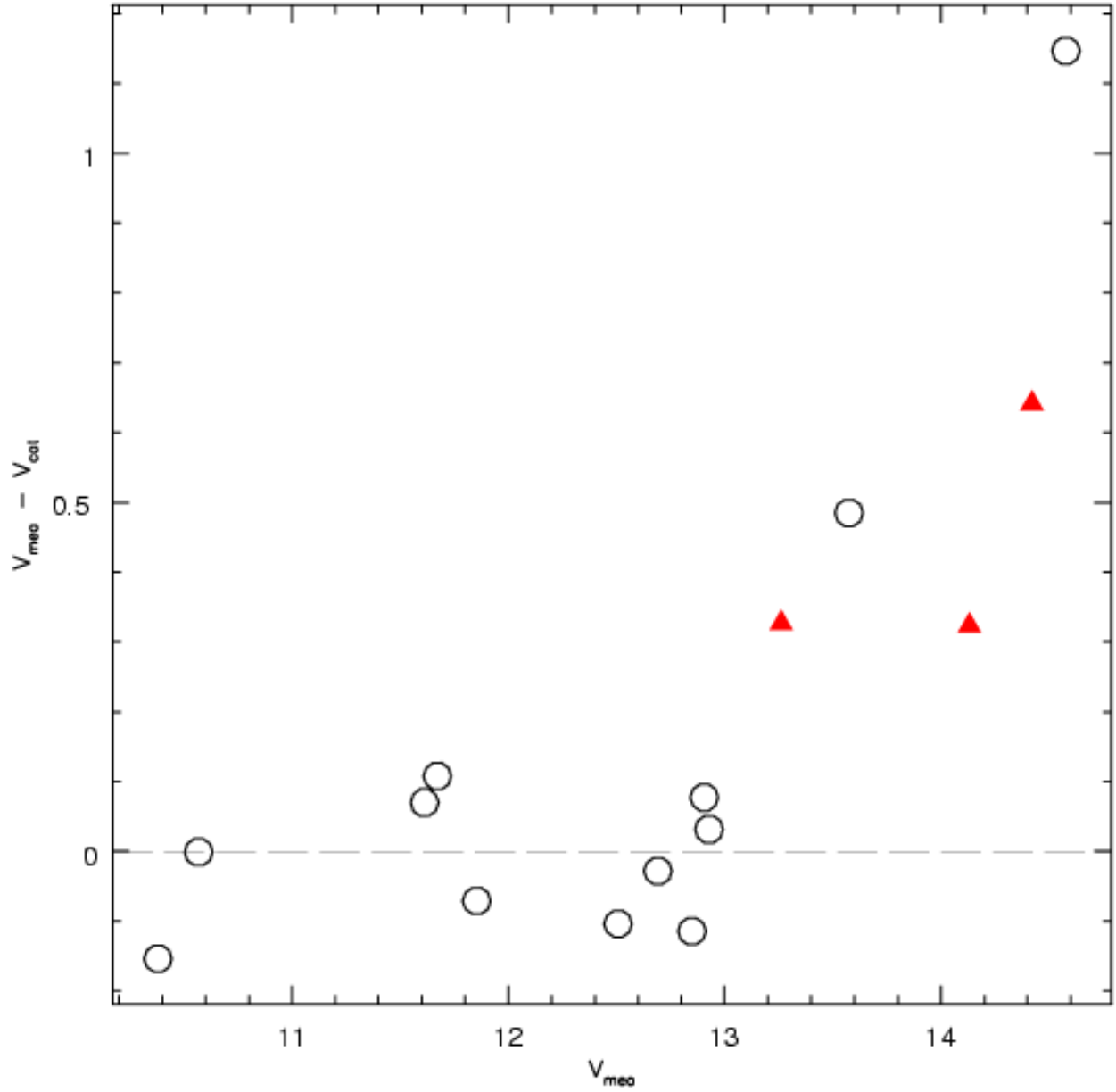


FIG. 8.— Differences between observed and calculated apparent V-band magnitudes of 15 asteroids observed by Krisciunas *et al.*. Asteroids having absolute magnitudes accurately determined (given to two decimal places in ASTORB catalog) are marked by circles. Triangles represent asteroids with poorly known absolute magnitudes (given to one decimal place). Dashed horizontal line serves as a visual aid, marking the position of zero magnitude difference.

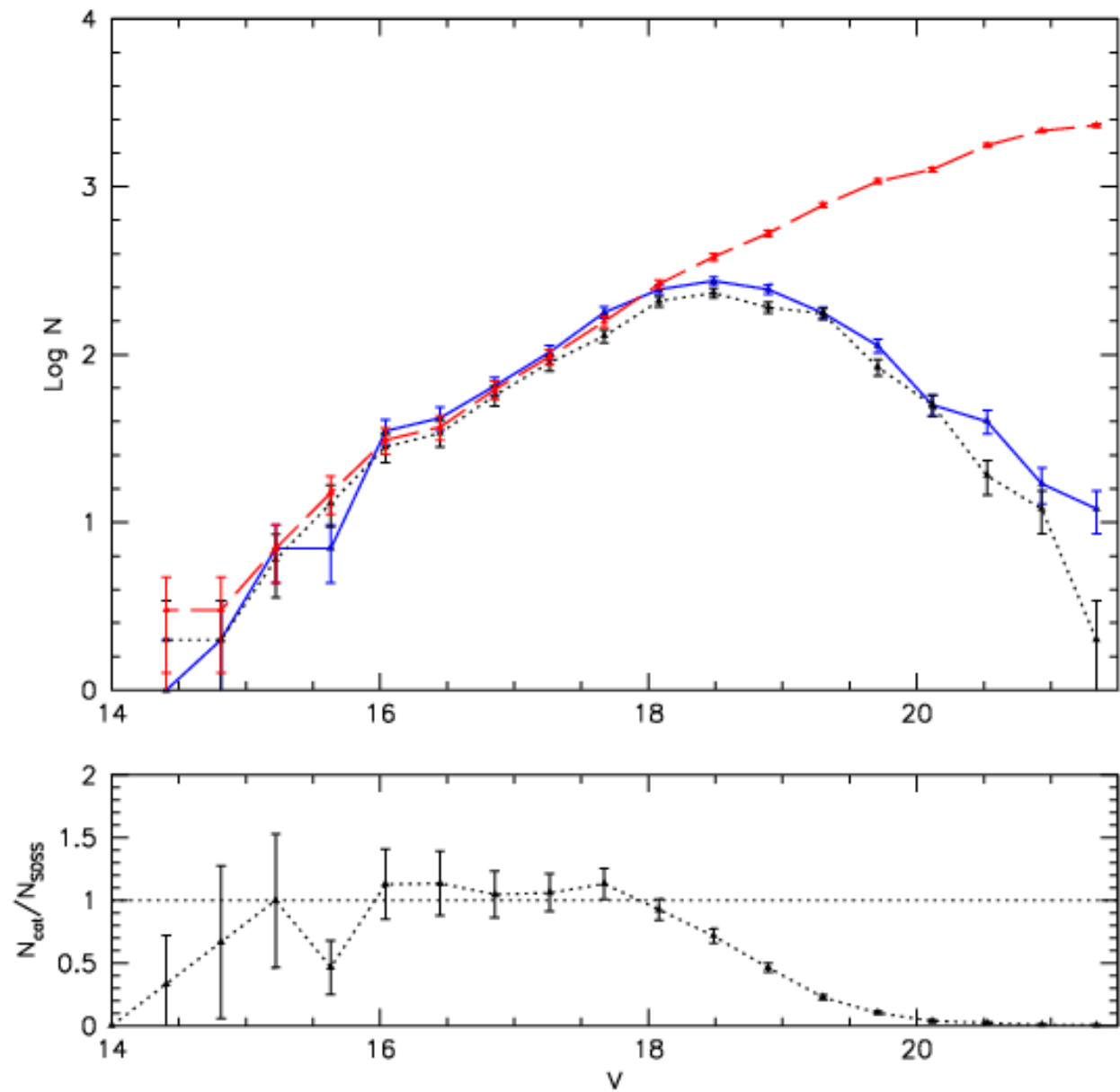


FIG. 9.— The apparent magnitude distributions as in Figure 4, except that the predicted magnitudes for ASTORB asteroids were offset by 0.41 mag (the full line). The shifted distribution implies SDSS completeness in agreement with that shown in Figure 3.

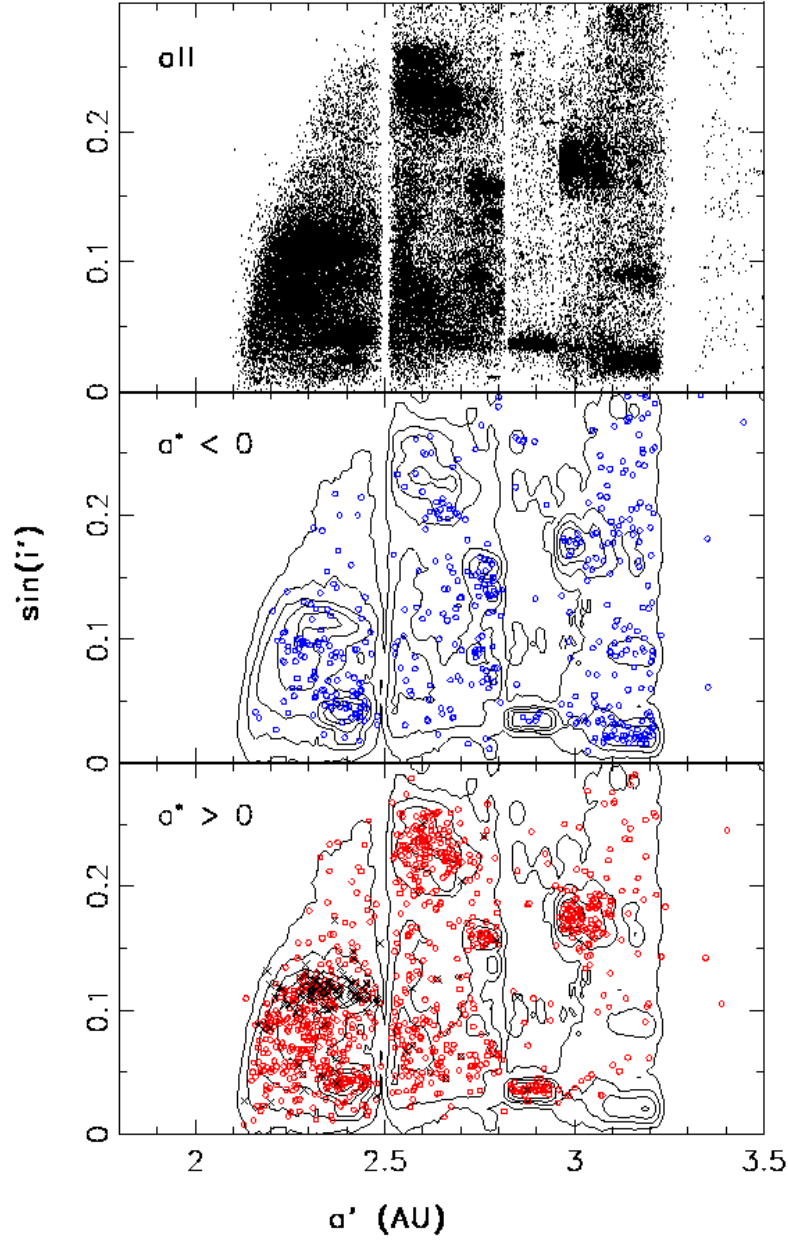


FIG. 10.— The distribution of asteroids in the $\sin(i')$ vs. a' plane. The top shows 67,917 asteroids with the known proper orbital elements, marked as dots. The remaining two panels show the same distribution as isodensity contours, with the matched asteroids shown by open circles. The middle panel shows blue asteroids ($a^* < 0$) and the bottom panel shows red asteroids ($a^* > 0$). A subset of red asteroids with $i^* - z^* < -0.25$ is shown by crosses; most are found around $a' \sim 2.2\text{--}2.5$ and $\sin(i') \sim 0.12$.

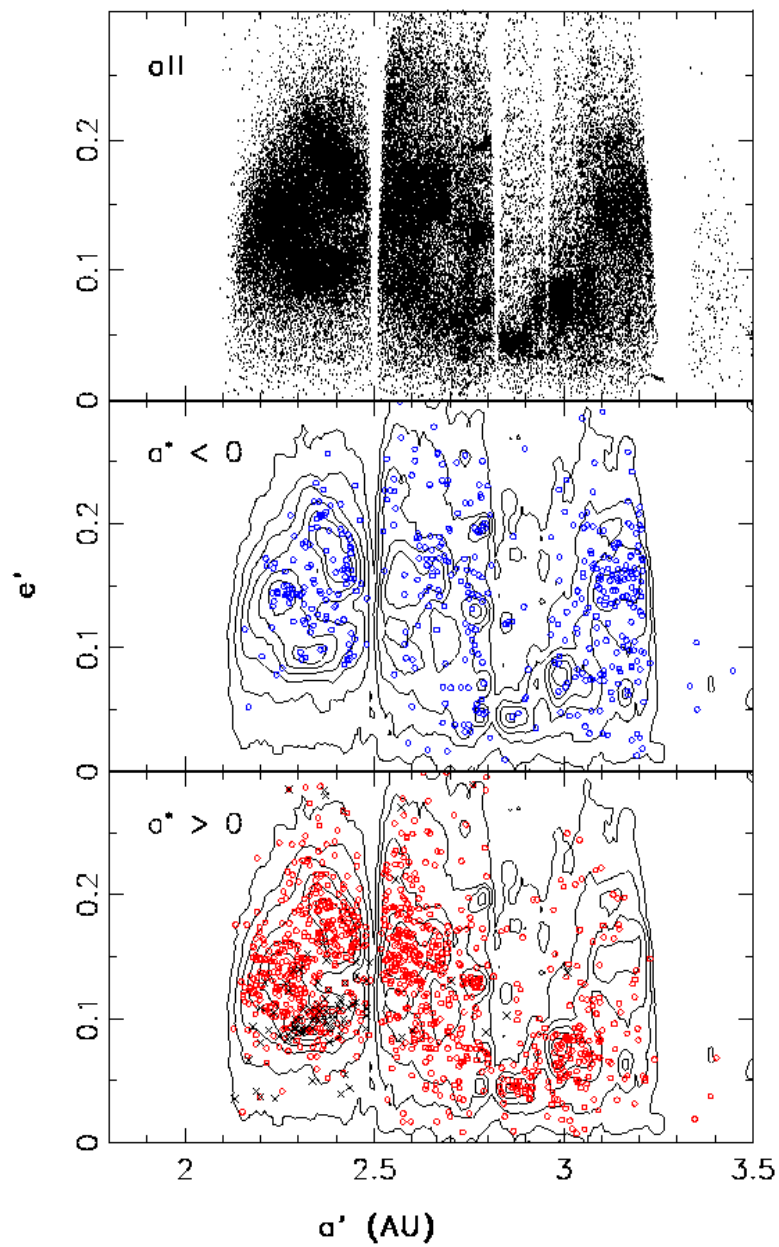


FIG. 11.— Same as the previous figure, except that the distribution of asteroids is shown in the e' (eccentricity) vs. a' plane.

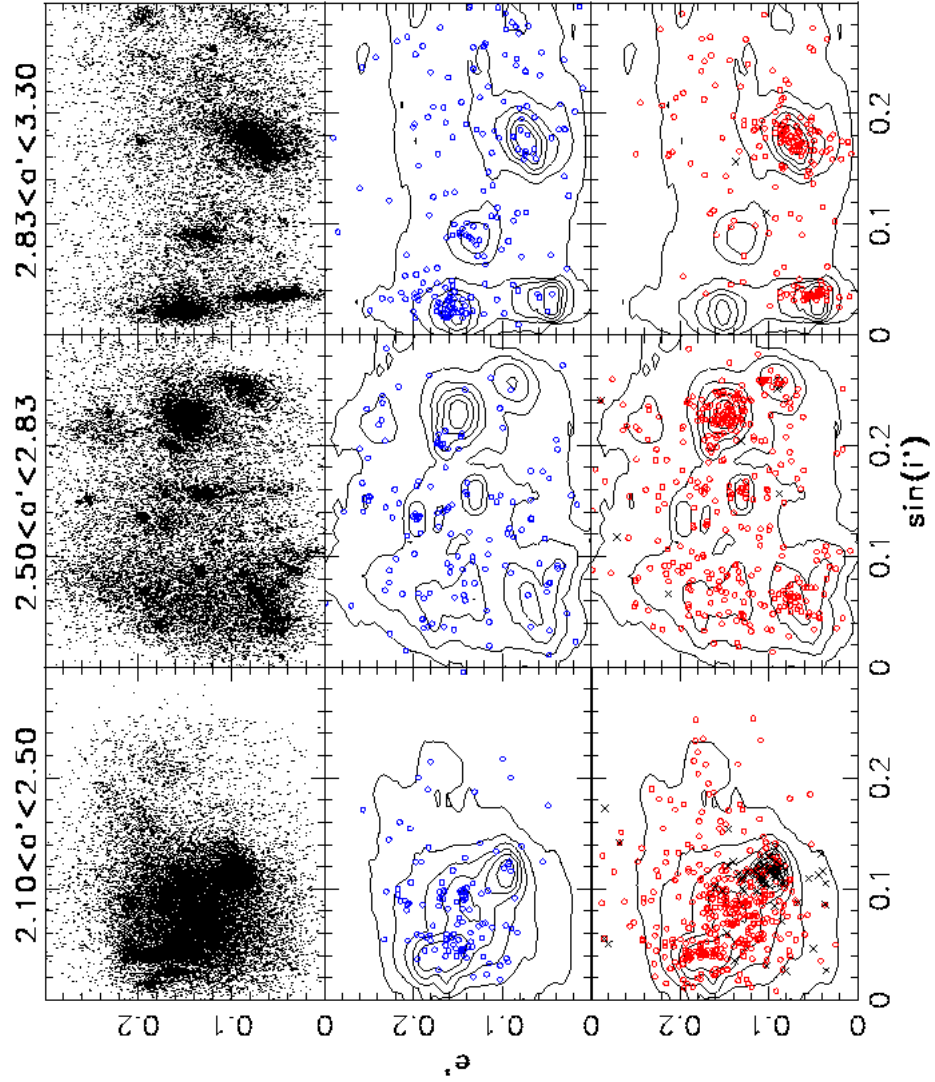


FIG. 12.— Same as the previous figure, except that the distribution of asteroids is shown in the e' vs. $\sin(i')$ plane, and for three ranges of the semi-major axis, a' , as marked on top.

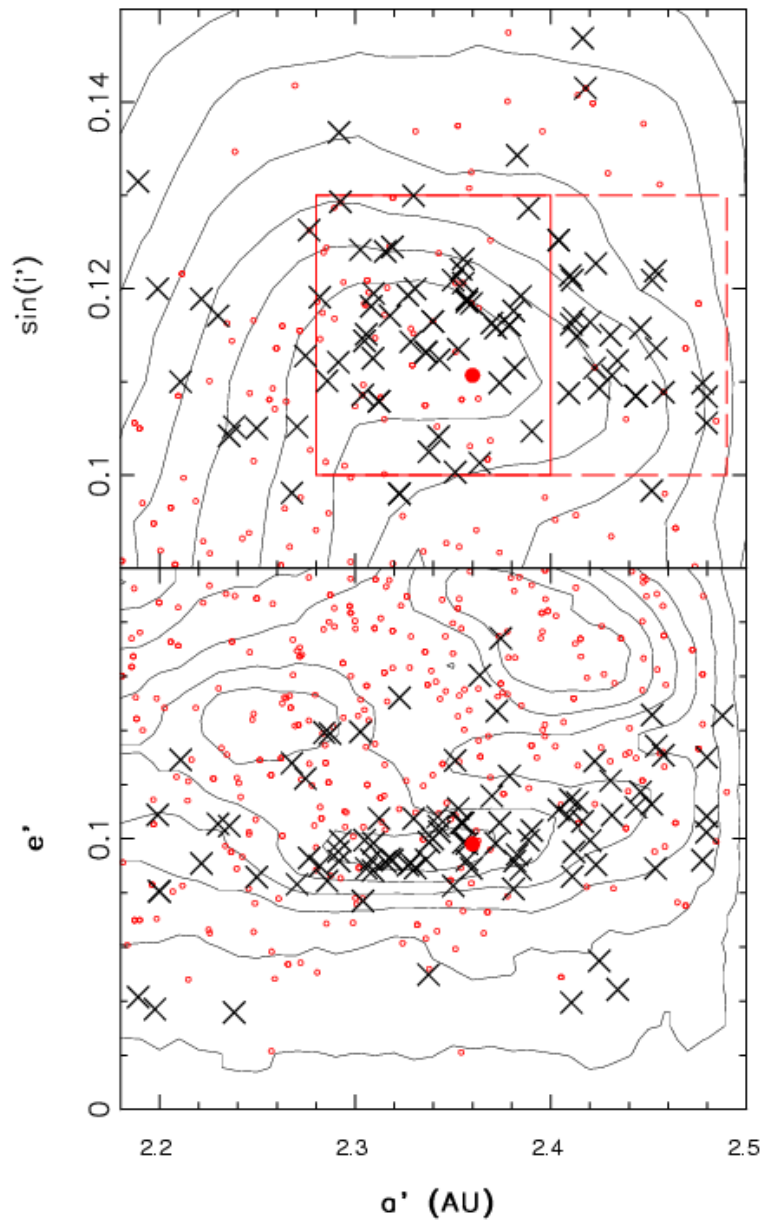


FIG. 13.— The regions from the previous three figures that contain the Vesta family. Only objects with $a^* > 0$ are shown; those with $i^* - z^* < -0.25$ are marked by crosses and others by circles. The boxes outline the core and the tail regions, as discussed by Zappala *et al.* (1995). The position of Vesta is marked by large dot.